defined as "that condition of temperature and pressure at which the partition K-value for gas-liquid equilibrium is equal to the partition K-value for gas-adsorbed phase equilibrium" as shown in Figure 1. The data indicated in Figure 1 are described in the prior article by Masukawa and Kobayashi (1968).

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Diffusivity Ratios In Fully Developed Turbulent Pipe Flow

R. H. NOTTER

Department of Chemical Engineering
Pennsylania State University, University Park, Pennsylvania 16802

and C. A. SLEICHER

Department of Chemical Engineering
University of Washington, Seattle, Washington 98195

In a recent R & D Note Hughmark (1974) has calculated some temperature profiles for the turbulent flow of liquids in pipes from a velocity profile equation of Brinkworth and Smith (1969) and the assumption that the eddy diffusivity for heat ϵ_H is equal to the eddy viscosity ϵ. Then using the data of several studies, mostly on liquid metals, he has plotted $\phi \equiv (T - T_c)$ calc$/(T - T_c)$ exp vs. y/R. The points scatter within 20% of unity, and the author states that this fact indicates equal diffusivities. We believe it does not for the reasons indicated.

The data of Sleicher, Awad and Notter (1973) were taken in a system where condensing steam was used to approximate a uniform wall temperature boundary condition, and the data were analyzed with this assumption. Hughmark's Equation (2) is valid for the boundary condition of uniform wall heat flux; it is invalid for uniform wall temperature. Because of a finite heat transfer resistance in the tube wall, the experiments of Sleicher, Awad, and Notter are actually somewhere between the extremes of uniform wall temperature and uniform wall heat flux, although the former is a better approximation as the Reynolds number decreases. In any case, we merely note here that Hughmark's temperature profiles were calculated on a uniform flux basis rather than a uniform wall temperature basis. However, this point is really peripheral and does not directly affect the conclusions reached below.

A more direct objection to the conclusion of equal diffusivities is that the calculated values of ϕ [see Figure 1 (Hughmark, 1974)] scatter about unity in a systematic way. In particular, the data of Buhr, Carr, and Balzhiser (1968) on NaK and Hg show $\phi > 1$ for all points whereas the data of Sleicher, Awad, and Notter on NaK show $\phi < 1$ for all points. In the latter paper we suggest that the difference between our results and those of Buhr, Carr, and Balzhiser (1968) is caused by swirl flow in their test section. Hughmark states that the Reynolds numbers he considers (of order 10^5) are low enough so that this effect is unimportant. Although the effect of the swirl on ϵ_H was greatest at high Reynolds number, swirl was present at all Reynolds numbers and, we believe, is the most reasonable explanation of the fact that ϕ is greater than unity for the data of Buhr, Carr, and Balzhiser. It is also significant that three different models of turbulence with widely differing assumptions and approximations (Jenkins, 1951; Deissler, 1952; Azer and Chao, 1960) all yield $\epsilon_H/\epsilon < 1$ for liquid metals.

Two other points are relevant with regard to Hughmark's calculations. First, Hughmark used the velocity profile expression of Brinkworth and Smith to calculate ϵ in the pipe core. This wholly empirical expression is an excellent analytical approximation to data, but it is not better than the data. In the core region we used data in tabular form given in the paper. That the profiles calculated by the two methods are very similar is a tribute to the expression of Brinkworth and Smith. Second, for fluids of even moderately high Prandtl number, like water, and for Reynolds number above about 50,000 the temperature profiles are so flat in the inner 90% of the pipe radius that calculations of ϵ_H are imprecise. For these conditions relatively large variations in ϵ_H (or ϵ_H/ϵ) near the pipe centerline will always yield values of ϕ that are close to unity, as found by Hughmark for water. Hence, Hughmark's calculations of ϕ from water data in the core of turbulent pipe flow are an insensitive test of ϵ_H/ϵ.

Finally, we should like to emphasize that although our data show ϵ_H/ϵ to be less than 1 for liquid metals, this is not true for other fluids in fully developed turbulent pipe flow. The diffusivity ratio is a function of position as well as of the Reynolds and Prandtl numbers. Experiments with various fluids of moderate Prandtl number under a variety of turbulent conditions have found values of ϵ_H/ϵ in the range 1.1 to 1.4, and a short review of this question is included in a recent paper by us (Notter and Sleicher, 1971).

In conclusion, we believe the contention of Hughmark (1974) that for fully developed pipe turbulence the ratio ϵ_H/ϵ is equal to one at all Prandtl numbers and at all radial positions is unwarranted. Not only is it in conflict with all theoretical analysis of ϵ_H/ϵ and with temperature profile measurements in liquid metals, but it also leads to errors in calculated heat transfer rates. For example, Notter and Sleicher (1972) show that eddy diffusivity profiles with ϵ_H/ϵ less than one at all radial positions yield Nusselt numbers that agree well with experimental data on carefully purified liquid metals in fully developed turbulent flow in smooth pipes for the boundary conditions of either uniform wall temperature or uniform wall heat flux. If ϵ_H/ϵ is assumed to be equal to one, similar calculations yield Nusselt numbers that exceed the data.

NOTATION

R = pipe radius
T = local time-averaged temperature
T_c = T at pipe center
y = radial distance from wall

Greek Letters

ϵ = eddy viscosity
ϵ_H = eddy diffusivity for heat
ϕ = $(T - T_c)$ calc/$(T - T_c)$ exp

Subscripts

calc = calculated
exp = experimental

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Reply to Notter and Sleicher Note

G. A. HUGHMARK

Ethyl Corporation
Baton Rouge, Louisiana 70821

The accuracy of the eddy diffusivity ratio for the core region of turbulent pipe flow is related to problems in obtaining both heat and momentum diffusivities. I question that these ratios are more accurate than the 20% maximum deviation shown by the prior R & D note for the core region.

Notter and Sleicher state that the contention of a unity ratio for eddy diffusivities of heat and momentum at all radial positions for fully developed pipe turbulence is unwarranted, is in conflict with theoretical analyses, and will lead to errors in calculated heat transfer rates. My note did not suggest a unity ratio for all radial positions but only for the core. Figure 1 shows data for $y/R >$ 0.075. Data are not shown for the wall region because there is an indication that the wall region ratio may not be unity (Hughmark, 1973). The models that yield a diffusivity ratio less than unity for liquid metals are based upon widely differing assumptions and approximations as stated by Notter and Sleicher, thus these do not appear to represent a theoretical analysis of greater accuracy than the broad range of assumptions and approximations. Table 1 shows Nusselt numbers calculated in accordance with Equations (19), (20), and (21) of my paper (1971) and Equation (1) of the note (1972) in comparison to the experimental values reported by Sleicher, Awad, and Notter. Thus these calculated values represent the assumption of an equal diffusivity ratio for the core region.

The three high velocity runs show agreement of about 10% which is excellent for liquid metal heat transfer data. The equal diffusivity ratio assumption does not lead to gross error as is implied by Notter and Sleicher.

TABLE 1. NUSSELT NUMBERS

Re	Notter et al. experimental *Nu*	Calculated *Nu*
26,000	6.8	7.9
52,000	9.1	10.4
79,000	10.5	12.8
106,000	13.5	14.8
203,000	20	20.7
302,000	29	31

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